Solvable for y :-

Ex. 1. $y = x\left\{\left(\frac{dy}{dx}\right) + \left(\frac{dy}{dx}\right)^2\right\}$ — (I)

$\Rightarrow y = x(P+P^2)$

$\Rightarrow y = Px + P^2x$ — (II)

$\Rightarrow P^2x + Px - y = 0$

Diff. both side w.r.t x in eqn (II)

$\Rightarrow \frac{dy}{dx} = \left(P + x\frac{dP}{dx}\right) + \left(P^2 + x.2P\frac{dP}{dx}\right)$

$\Rightarrow \cancel{P} = \cancel{P} + x\frac{dP}{dx} + P^2 + 2xP\frac{dP}{dx}$

$\Rightarrow -P^2 = x(1+2P).\frac{dP}{dx}$

$\Rightarrow -\frac{dx}{x} = \frac{(1+2P)}{P^2}dP$

$\Rightarrow \log x = -\left[\int\frac{dP}{P^2} + \int\frac{2}{P}.dP\right]$

$= \log x = -\left(\frac{-1}{P} + 2.\log P\right)$

$\Rightarrow \log x = \frac{1}{P} - \log P^2 + \log c$

$\Rightarrow \log x + \log P^2 - \log c = \frac{1}{P}$

$\Rightarrow \log\frac{P^2x}{c} = \frac{1}{P} \quad \Rightarrow \frac{P^2x}{c} = e^{1/P}$

$\Rightarrow P^2 = x^{-1}c\,e^{1/P}$

$\Rightarrow x = cP^{-2}e^{1/P}$ — (III)

The P-eliminant of (I) & (III) will give soln

2. $e^y = p^3 + p$ —— (I)

Diff. both side w.r.t x

$\Rightarrow e^y \cdot \frac{dy}{dx} = 3p^2 \cdot \frac{dp}{dx} + \frac{dp}{dx}$

$\Rightarrow e^y \cdot p = (1+3p^2)\frac{dp}{dx}$

$\Rightarrow (p^3+p)p = (1+3p^2)\frac{dp}{dx}$

$\Rightarrow (p^4+p^2) = (1+3p^2)\frac{dp}{dx}$

$\Rightarrow dx = \frac{1+3p^2}{p^2(p^2+1)} \cdot dp = \left\{\frac{1}{p^2} + \frac{2}{(p^2+1)}\right\} dp$

$\Rightarrow x = \frac{-1}{p} + 2\tan^{-1}p$

$\Rightarrow x = 2\tan^{-1}p - \frac{1}{p} + c$ —— (II)

~~from (I) & (II)~~

The p-eliminant of (I) & (II) will give the required solution.

3. $y = 2px + p^2$ —— (I).

Diff. both side w.r.t x

$\Rightarrow \frac{dy}{dx} = 2p + 2x \cdot \frac{dp}{dx} + 2p \cdot \frac{dp}{dx}$

$\Rightarrow p = 2p + 2x \cdot \frac{dp}{dx} + 2p \cdot \frac{dp}{dx}$

$\Rightarrow p + 2(x+p)\frac{dp}{dx} = 0$

$\Rightarrow 2(x+p)\frac{dp}{dx} = -p$

$\Rightarrow \frac{dp}{dx} = \frac{-p}{2(x+p)}$

$\Rightarrow \frac{dx}{dp} = \frac{2(x+p)}{-p}$

$\Rightarrow \frac{dx}{dp} = -\frac{2}{p}.x - 2$

$\Rightarrow \frac{dx}{dp} + \frac{2}{p}.x = -2$

It is linear in x.

Here, I.F $= e^{2\int\frac{dp}{p}} = e^{2\log p} = e^{\log p^2} = p^2$

Hence, the solution is

$x.p^2 = \int -2p^2\,dp$

$\Rightarrow xp^2 = -2.\frac{p^3}{3} + c$

$\Rightarrow x = -\frac{2}{3}p + cp^{-2}$ —— (II)

Now, from (I)

$y = 2p(-\frac{2}{3}p + cp^{-2}) + p^2$

$y = -\frac{4}{3}p^2 + \frac{2c}{p} + p^2 = \frac{2c}{p} - \frac{1}{3}p^2$ — (III)

The eq^n (II) and (III) Constitute the required sol^n

4. $y = (1+p)x + ap^2$ —— (I)

$\Rightarrow \frac{dy}{dx} = (1+p) + x.\frac{dp}{dx} + a.2p\frac{dp}{dx}$

$\Rightarrow p = 1+p + (2ap+x)\frac{dp}{dx}$

$\Rightarrow 1 + (2ap+x)\frac{dp}{dx} = 0$

$\Rightarrow (2ap+x)\frac{dp}{dx} = -1$

$\Rightarrow \frac{dp}{dx} = \frac{-1}{2ap+x}$

$\Rightarrow \frac{dx}{dp} = -x - 2ap$

$\Rightarrow \frac{dx}{dp} + x = -2ap$

Here, $I.F = e^{\int dp} = e^p$

Hence, the sol^n is

$x.e^p = \int -2ap \cdot e^p \, dp$

$\Rightarrow x.e^p = -2a\int pe^p dp$

$\Rightarrow xe^p = -2a[pe^p - e^p] + c$

$\Rightarrow xe^p = -2a.e^p(p-1) + c$

$\Rightarrow x = -2a(p-1) + ce^{-p}$

$\Rightarrow x = 2a(1-p) + ce^{-p}$ —— (II)

The p-eliminant of (I) & (II) will give the required solution

Q. $y + px = x^4 p^2 \Rightarrow y = -px + x^4 p^2$

Diff. both side w.r.t x

$$\Rightarrow \frac{dy}{dx} = -p - x\frac{dp}{dx} + x^4 \cdot 2p\frac{dp}{dx} + 4p^2x^3$$

$$\Rightarrow p = -p - x\frac{dp}{dx} + 2px^4\frac{dp}{dx} + 4p^2x^3$$

$$\Rightarrow 2p + x\frac{dp}{dx} - 2px^4\frac{dp}{dx} - 4p^2x^3 = 0$$

$$\Rightarrow 2p(1-2px^3) + x\frac{dp}{dx}(1-2px^3) = 0$$

$$\Rightarrow \left(2p + x\frac{dp}{dx}\right)(1-2px^3) = 0$$

$$\Rightarrow 2p + x\frac{dp}{dx} = 0$$

$$\Rightarrow x\frac{dp}{dx} = -2p$$

$$\Rightarrow \frac{dp}{p} = -2 \cdot \frac{dx}{x}$$

$$\Rightarrow \log p = -2\log x + \log c$$

$$\Rightarrow \log p + \log x^2 = \log c$$

$$\Rightarrow \log px^2 = \log c$$

$$\Rightarrow px^2 = c$$

$$\Rightarrow p = \frac{c}{x^2}$$

from ① $y = -\frac{c}{x^2} \cdot x + x^4 \cdot \frac{c^2}{x^4} \Rightarrow y + \frac{c}{x} - c^2 = 0$

$$\Rightarrow xy - c^2x + c = 0$$

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